

Cosmological anomalies and exotic smoothness structures

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Abstract

It seems to be generally accepted that apparently anomalous cosmological observations, such as accelerating expansion, etc., necessarily are inconsistent with standard general relativity and standard matter sources. Following the suggestions of Śladrkowski, we point out that in addition to exotic theories and exotic matter there is another possibility. We refer to exotic differential structures on \mathbb{R}^4 which could be the source of the observed anomalies without changing the Einstein equations or introducing strange forms of matter.

Recent cosmological observations have been interpreted as indicating an accelerating expansion of the universe [1], [2]. This conclusion in turn has led to speculations that either or both of the following must be considered:

1. Einstein's equations in their original, purely metric, form in four dimensions are incorrect, or,
2. the matter tensor contains “exotic” sources, such as dark energy, quintessence, etc.

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The recent literature is replete with speculations along these lines. We cite just a few representatives: [3], [4], [5], [6], [7], [8], etc.

In this paper we want to revisit the suggestion of Ślaskowski[9] that there is another possibility based on recent mathematical discoveries in differential topology. Specifically,

3. the coordinates (t, r) of observational cosmology may not be smoothly extendible indefinitely into the past.

If 3. is valid then the standard extrapolation of earth based observations to distant phenomena may not be justified.

The reason for raising the conjecture 3. lies in the discoveries in differential topology of the existence of non-standard, or exotic, global smoothness (differential) structures on topologically trivial spaces such as \mathbb{R}^4 , or $\mathbb{R}^1 \times S^3$. For reviews of the subject aimed at the physics audience see [10], [11], [12]. Let us begin here by simply stating a strikingly counter intuitive fact as well established mathematically:

Theorem 1 *There exist global smoothness structures on topological \mathbb{R}^4 which are not diffeomorphic to the standard one. We label such manifolds \mathbb{R}_{Θ}^4 .*

Thus, we can label points \mathbb{R}_{Θ}^4 with global *topological* coordinates, (t, x, y, z) . However, to do calculus we need a *differential (smooth) structure* defined by a family of coordinate patches with smooth coordinate transformations in their overlap. Such a family of coordinate patches defines a smooth structure. One obvious one on \mathbb{R}^4 is the “standard” one defined by one coordinate patch with smooth coordinates identical to the global topological ones. It has been known for some time that *any* smooth structure is diffeomorphic (equivalent) to the standard one for all \mathbb{R}^n , $n \neq 4$. However, the conjecture that this would also be true for physically critical case of $n = 4$ remained unsettled until the 1980’s when pioneering work by Donaldson, Freedman, Gompf, et al., established Theorem 1. Thus the statement of the theorem is that not all of (t, x, y, z) can be global smooth functions in terms of this exotic structure.

For our purposes a remarkable feature of these exotic \mathbb{R}_{Θ}^4 ’s is that each of them contains a compact set that cannot be contained in the interior of any smoothly embedded S^3 . See, for example, the discussion in pages 366ff of Gompf and Stipsicz,[13]

Theorem 2 *For some \mathbb{R}_{Θ}^4 , there exist global topological coordinates (t, x, y, z) and numbers $R_1 < R_2$ such that the spheres, S_{R_0} , defined by $t^2 + x^2 + y^2 + z^2 =$*

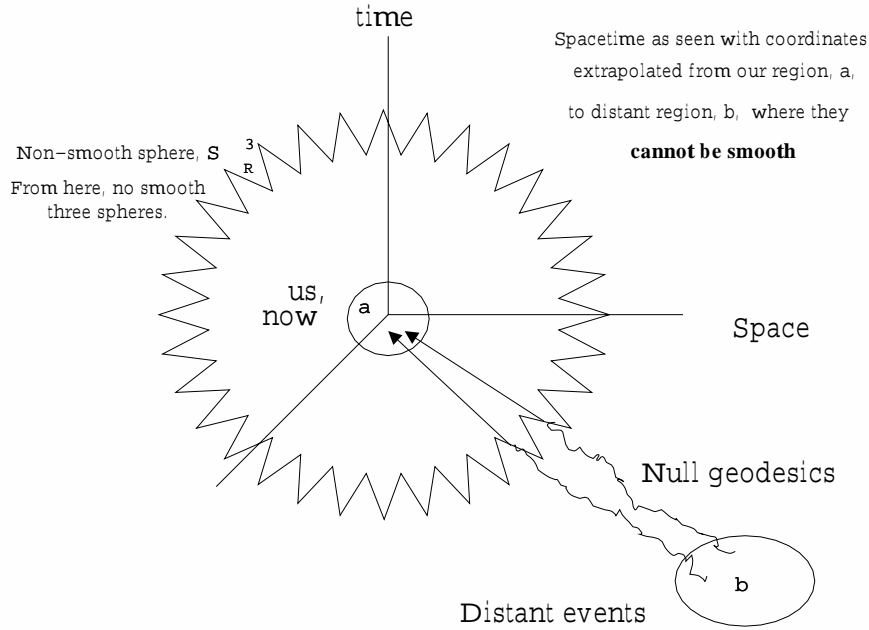


Figure 1: Exotic spacetime with non-smooth $S^3_{R_2}$. Coordinates cannot be smoothly continued past limiting topological $S^3_{R_2}$.

R_0^2 are smooth for $R_0 < R_1$, but are not smooth³ for any $R_0 \geq R_2$. Choose one such, say M , for our spacetime model.

We can thus state that for M

Theorem 3 *We can choose two sets, a and b in M such that both cannot be included in one smooth coordinate patch in any diffeomorphic presentation of M*

Now look at null geodesics between points in these two sets (see figure) and attempts to interpret information received in a from b in terms of the a priori assumption that one coordinate patch including the pair exists.

³By “not smooth” we mean “not smoothly embedded.”

In looking at the figure, we must point out that the light rays, null geodesics, are smooth *locally*. The reason they are represented as non-smooth in this drawing is to point out that an astronomer trying to draw such a figure based on his observations in a alone would be forced to use non-smooth lines since the local smooth coordinates in a can *not* be smoothly continued to b . In other words, no smooth image such as in this figure can be drawn for our $M = \mathbb{R}_\Theta^4$. The null geodesics from b will still be smooth and well behaved throughout their length, and the Einstein equations satisfied with normal matter. However, it will be incorrect to assume that we can extrapolate from these incoming geodesics in a information about b *because we do not know the non-trivial transition function between the smooth coordinate patches linking the two sets*.

More specifically, in observational astronomy it is generally assumed that the metric can be written in the FRW form

$$ds^2 = -dt^2 + a(t)^2 d\sigma_3^2, \quad (1)$$

where the spatial three metric is usually expressed in spherical coordinates in a form depending on assumptions of isotropy and homogeneity. The associated topology is thus $\mathbb{R}^1 \times M^3$ for some three-manifold, M^3 . In the standard models the three metric is one of the three constant curvature ones, each containing a “radial” coordinate⁴, r . Because of isotropy, the incoming geodesics are described globally (modulo the proviso in the footnote) by differential equations involving r, t only. However, if M is as described in Theorem 3 and the figure, these may not be globally smooth. Hence the actual metric would have to be expressed in terms of more than one r, t coordinate region, and information extracted from the coordinate overlaps. Unfortunately, because the present mathematical technology does not provide us with an effective coordinate patch structure, more explicit statements than this cannot now be made. Nevertheless, the assumption that we can extrapolate information coming from incoming light rays back in time and out in space as if these geodesics would act as a radial type of coordinate system when indefinitely extended into their past is not valid if M is used as a spacetime model. We should also note that although we have discussed only the \mathbb{R}_Θ^4 (which is actually $\mathbb{R}^1 \times_\Theta \mathbb{R}^3$) we could equally have chosen an exotic $\mathbb{R}^1 \times_\Theta S^3$.

⁴Of course in the spherical case the “radial” coordinate is not indefinitely continuable because it is essentially an angular one. However, this is not the sort of coordinate anomaly we are addressing here and can certainly be accommodated in standard models.

A simple analogy is provided by gravitational lensing phenomena. Here we see two incoming null geodesics arriving at earth from different directions. However, the possibility that in some reasonable situations they cannot be extrapolated backward as “good” radial coordinates because they have been focused by the gravitational lens effect of an intervening massive object has been widely discussed and generally accepted as viable. Thus the extrapolation of the different angle data for the two incoming geodesics to different sources is incorrect.

Statement 1 (Gravitational lensing analogy) *Null geodesics arriving from different angles may intersect in the past because of gravitational curvature caused by intervening mass and thus may not be extrapolated back as good radial coordinate lines.*

What we are proposing here is more radical, of course, but just as viable in the sense that we know of no physical principles to exclude it, and it could lead to an understanding of apparent anomalous distant time behavior without introducing exotic theories or matter, just exotic smoothness of the spacetime manifold model.

Statement 2 (Exotic structures) *Null geodesics arriving from distant sources may not be extrapolated back as good radial coordinate lines because of intervening coordinate patch transformations caused by global exotic smoothness.*

In summary, what we want to emphasize is that without changing the Einstein equations or introducing exotic, yet undiscovered forms of matter, or even without changing topology, there is a vast resource of possible explanations for recently observed surprising astrophysical data at the cosmological scale provided by differential topology.

While it is true that at this stage of development of the mathematical technology it is not possible to give explicitly the coordinate patch overlap functions, research along these lines is being actively pursued. Furthermore, Śladrkowski [14], has shown that it is possible to relate isometry groups (geometry) to differential structures in some cases.

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